VELOCITY OF SOUND IN WATER AS A FUNCTION OF TEMPERATURE AND PRESSURE

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ABSTRACT. It is shown that the variation of velocity of sound (u) in water with temperature (t) at different pressures can be represented by $u = a+bt+ct^2$. Using Smith and Lawson's data, maximum velocity temperatures at different pressures have been calculated. The maximum velocity temperatures appear to increase with the increase in pressure but at very high pressures the data in inconclusive.

INTRODUCTION

Water is known to show an abnormal behaviour in many of its physical properties as shown by Partington (1951). And that is also true for the velocity of sound in it. It is known that all pure liquids show a linear variation of sound velocity with temperature and a negative temperature coefficient Water, on the other hand, displays a maximum velocity (at about 74°C at atmospheric pressure) and the temperature coefficient changes from positive to negative at this temperature (which for the sake of abbreviation we shall call as 'maximum velocity temperature'). Frequently this anomalous behaviour of water is attributed to association, yet methyl alcohol and several other associative compounds do not have a variation similar to that of water.

In recent years considerable attention has been paid to the measurement of sound velocity in water. Willard (1947) has shown that the velocity of sound can be represented as

$$u=1557-0.0245\,(74-t)^2$$

which gives a maximum velocity at 74°C. Greenspan, Tschiegg and Breckenridge (1956) found the maximum velocity temperature to be 73.95°C. On the other hand Salceanu (1957) made measurements of sound velocity at temperatures between 27°C and 81°C at a frequency of 1315 cycles per second and found that the maximum velocity is at about 62°C.

Pancholy (1953) has investigated the velocity of sound in heavy water. Lagemann, Gilley and McLeroy (1953) have determined the velocity of ultrasonics in supercooled water and heavy water. Highly accurate measurements (accuracy one part in 30,000) for the sound velocity in water from 0°C to 100°C have been recently reported by Greenspan and Tschiegg (1957) who represent their results by a fifth degree polynomial.

Recently determination of sound velocity in water has also been made at high pressures. Holton (1951) has reported measurements on the velocity of sound in water as a function of pressure up to 6000 Kg/cm² at two different temperatures. Smith and Lawson (1954) using an ultrasonic echo technique have carried on similar measurements at hydrostatic pressures varying up to 9600 Kg/cm². Martin (1957) has determined the velocity of high frequency sound waves in distilled water and in standard sea water at 25°C between 0 and 1000 atm. pressures using a pulse technique.

The measurements of Holton and those of Smith and Lawson show an important discrepancy as regards the behaviour of the maximum velocity of sound as a function of temperature as the pressure is increased. Holton concludes from his measurements that this temperature decreases with increasing pressure while Smith and Lawson finds an opposite behaviour. The latter authors have given graphs (their figure 4) showing that the maximum velocity temperature increases gradually with increasing pressure, though no precise analysis of the data is given.

In this paper we have examined Smith and Lawson's data by analytical methods to find the exact behaviour of maximum velocity temperature and maximum velocity with increase in pressure.

It was found that the variation of the velocity of sound (u) with temperature at different pressure can be adequately represented by

$$u = a + bt + ct^2$$

where (t) is the temperature in Centigrade degrees and a, b, c are constants.

Smith and Lawson's values at six different pressures were used to evaluate the constants a, b and c by the method of least squares. The values thus determined are produced in Table I. The calculated and experimentally observed values of u are shown in Tables II to VII. The maximum velocity temperature is given by $-b/2c$. Its calculated values as well as maximum velocities are recorded in Table VIII.

TABLE I

No	Pressures Kg/cm²	a	b	c
1	1	1407.546	4.24663	−.0295928
2	435	1492.542	3.62012	−.0218743
3	1039	1605 788	3.21895	−.0181910
4	5544	2262.727	1.49529	−.00785982
5	7370	2484 108	0.068427	+.00144287
6	9410	2617.304	1.32011	−.0073539

TABLE II

For pressure of 1 Kg/cm^2

No.	t°C	u, Calculated	m/sec. Observed	Difference u(calc) − u(obs)
1	0	1407.5	1403	+4.5
2	22.5	1488.1	1488	+0.1
3	24.2	1493.0	1494	−1.0
4	26.6	1499.6	1504	−4.4
5	27.0	1500.6	1505	−4.4
6	27.6	1502.2	1504	−1.8
7	45.4	1539.3	1539	+0.3
8	55.1	1551.7	1547	+4.7
9	65.5	1558.7	1555	+3.7
10	74.7	1559.6	1557	+2.6
11	83.2	1556.0	1557	−1.0
12	93.8	1545.5	1549	−3.5

TABLE III

For pressure of 435 Kg/cm^2

No.	t°C	u, Calculated	m/sec. Observed	Difference u(calc) − u(obsd)
1	22.5	1562.9	1563	−0.1
2	57.6	1628.5	1628	+0.5
3	66.9	1636.8	1637	−0.2
4	77.0	1641.6	1642	−0.4
5	86.7	1642.0	1642	0.0
6	96.5	1638.2	1638	+0.2

TABLE IV

For pressure of 1039 Kg/cm^2

No.	t°C	u, calculated	m/sec. observed	Difference u(calc)$-u$(obsd)
1	26.5	1678.3	1677	+1.3
2	44.2	1712.5	1714	−1.5
3	55.4	1728.3	1729	−0.7
4	63.4	1736.7	1737	−0.3
5	70.5	1742.3	1742	+0.3
6	75.5	1745.1	1745	+0.1
7	80.4	1747.0	1746	+1.0
8	88.1	1748.2	1748	+0.2
9	96.4	1747.0	1747	0.0
10	104.6	1743.5	1744	−0.5

TABLE V

For pressure of 5544 Kg/cm^2

No.	t°C	u, calculated	m/sec. observed	Difference u(calc)$-u$(obsd)
1	0.0	2262.7	2264	−1.3
2	22.6	2292.5	2290	+2.5
3	57.4	2322.7	2324	−1.3
4	66.5	2327.4	2327	+0.4
5	76.3	2331.1	2331	+0.1
6	85.9	2333.2	2335	−1.8
7	96.1	2333.8	2333	+0.8
8	103.3	2333.3	2334	−0.7
9	103.4	2333.3	2332	+1.3

TABLE VI

For pressure of 7370 Kg/cm²

No.	t°C	u, calculated	m/sec. observed	Difference u(calc) − u(obsd)
1	19.1	2485.9	2485	+0.9
2	42.8	2489.7	2492	−2.3
3	51.1	2491.4	2491	+0.4
4	63.4	2494.2	2495	−0.8
5	69.7	2495.9	2495	+0.9
6	84.0	2500.0	2497	+3.0
7	95.6	2503.8	2506	−2.2

TABLE VII

For pressure of 9410 Kg/cm²

No.	t°C	u, calculated	m/sec. observed	Difference u(calc) − u(calc)
1	56.5	2668.4	2671	−2.6
2	62.4	2671.0	2669	−2.0
3	70.2	2673.7	2672	+1.7
4	82.7	2676.2	2676	+0.2
5	97.4	2676.1	2678	−1.9
6	98.4	2676.0	2676	0.0
7	128.9	2665.3	2664	+1.3
8	129.0	2665.2	2666	−0.8

TABLE VIII

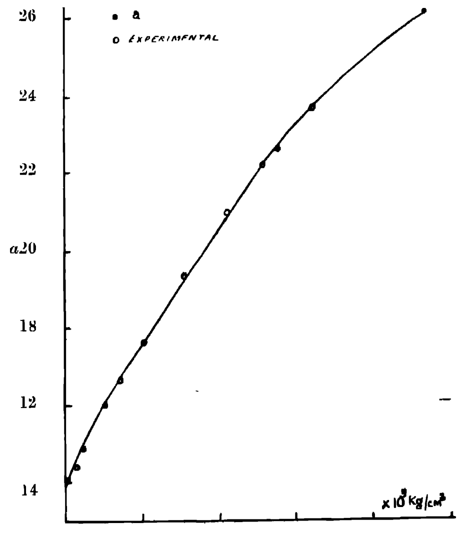
No.	Pressure Kg/cm²	Maximum Velocity Temperature °C	Maximum Velocity
1	1	71.75	1559.9
2	435	82.75	1642.3
3	1039	88.48	1748.2
4	5544	95.12	2333.8
5	7370	×	×
6	9410	89.76	2676.5

DISCUSSION

Though Smith and Lawson (1954) have not given the uncertainty in their values it appears that the values differ by 2 m/sec from their values given in figure 6. Tables 2-7 show that the average difference in the calculated and observed values of the sound velocity is 1.4 m/sec. This is within the limits of experimental error postulated above

The results for pressure of 7370 Kg/cm^2 do not show a maxima because it is found that the constant c in this instance is positive whereas for other pressures its values are negative. Also the values of a and b for this pressure do not follow the general trend in thir variation with pressures as seen in Table I. It is, therefore, suspected that the experimental values for this pressure are not sufficiently accurate

The constant a represents the velocity of sound at 0°C Its variation with pressure is seen in Table 1 and figure 1. Fortunately, Smith and Lawson have



Pressure

Fig. 1. Shows variation of a with pressure.

experimentally determined the variation of velocity with pressure at 0°C. Some of their experimental points are shown in figure 1 by open circles. As expected both the above sets of points lie on a smooth curve. Point corresponding to a pressure of 7370 Kg/cm² is not considered.

Behaviour of constants b and c is shown in Figure 2. The values of b and c are observed to decrease smoothly with pressure.

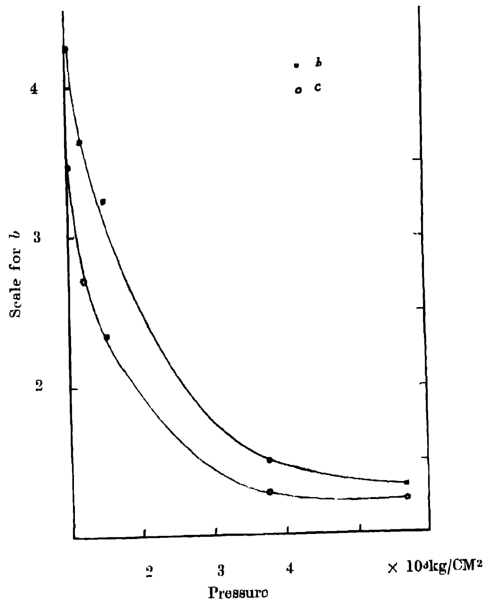


Fig. 2. Shows variation of constants b and c with changes in pressures.

Maximum velocity temperature as determined with the above theoretical expression at a pressure of 1 Kg/cm² is 71.75°C. The value determined by Greenspan *et al* (1956) at atmospheric pressure i.e. 1.03 Kg/cm² is 73.95°C. There is thus a difference of 2.2°C The discrepancy at other higher pressures is bound to be greater in magnitude. The maximum velocity temperature increases with pressure up to 5544 Kg/cm² but the value at 9410 Kg/cm² is found to be lower than the previous value. It has been pointed out that the value at 7370 Kg/cm² is not sufficiently accurate. The temperature gradient at higher pressures is very small (Table 6, 7) and hence even small error in measurements can seriously vitiate the position of the maxima. The experimental uncertainty of ± 2 m/sec at higher pressures can be responsible for a variation of about 5°C in the value of maximum velocity temperature from the correct value.

The trend of maximum velocity temperature is to increase with pressure, yet we do not consider it safe to draw any precise conclusion from this fact regarding the numerical variation. Perhaps the value at 5544 Kg/cm^2 is a bit too large and that at 9410 a bit too small.

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